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# APPLICATION OF CUBIC BIPOLAR INTUITIONISTIC FUZZY SETS TO MCDM USING AGGREGATION OPERATORS WITH PRIORITY DEGREES

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**ABSTRACT:**
We introduce the operators: "cubic bipolar intuitionistic fuzzy prioritized weighted averaging aggregation operators (CBIFPWA)", "cubic bipolar intuitionistic fuzzy prioritized weighted geometric aggregation operators (CBIFPWG)", "cubic bipolar intuitionistic fuzzy prioritised averaging aggregation operators with priority degrees (CBIFPDA)" and "cubic bipolar intuitionistic fuzzy prioritised geometric aggregation operators with priority degrees (CBIFPDG)" in this work. Additionally, we provide a better score function as well as accuracy function for comparing the cubic bipolar intuitionistic fuzzy numbers (CBIFNs) and examine the utility and efficacy of these operators in MCDM issues.
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## INTRODUCTION:

In MCDM, several people choose at the same time from the possibilitiesin front of them. Because of the growing complexity of the environment in choice analysis and the circumstances themselves, decision makers are occasionally persuaded by numerical information to provide decision making information. As a result, Atanassov [1] demonstrated the intuitionistic fuzzy set (IFS) by extending the Zadeh's [2] fuzzy set. The IFS was further extended by Atanassov and Gargov [3] and came up with the concept of interval-valued intuitionistic fuzzy sets (IVIFS), and outlined the fundamental IVIFS algorithm. Inevitably, both IFS and IVIFS successfully addresses the real-world problems that allow for ambiguity; nonetheless, further research is necessary to find a substitute that both possesses IFS and IVIFS**.**

In order to cope with this problem, Zhang [4, 5] presented an entirely new fuzzy set which was represented as Bipolar Fuzzy Set (BFS). Wei et al. [6] put forward the idea of interval-valued bipolar fuzzy set (IVBFS) in order to broaden the scope of BFS. The notion of a bipolar intuitionistic fuzzy sets, which Ezhilmaran et al. [7] put forward as an extended form of bipolar fuzzy set. With the goal of improving the domain of membership degrees in fuzzy sets, Jun et al. [8] created the idea of a Cubic set and its related operations, which is an extended form of a FS and IVFS. Kaur and Garg [9] expanded the idea of the CFS to create the cubic Intuitionistic Fuzzy Sets (CIFS), where the degree of rejection was also included in the analysis. One effective way to discern between options is to use the aggregation operators method under various operations.These operators met a number of significant requirements and are beneficial in a variety of domains, including as business, finance, economics, etc., We frequently encounter situations, though, in which there is a clear prioritizing link between the points need to be combined. Using IFSs, Yu and Xu [10]proposed priority aggregation operators.

In order to address a wide range of challenging issues, the researchers have effectively implemented numerous optimization strategies. When it comes to accurately describing the occurrence of evaluations or assessments, the concept of a simple BIFS falls short due to its limited informational scope and lack of capacity to capture the occurrence of ambiguity and uncertainty, particularly in situations where decision-making involves delicate cases. In a similar manner, the notion of an interval-valued bipolar intuitionistic fuzzy set is likewise inadequate for revealing the experts opinions that depend on the characteristics of the options. In order to do this, we had introduced a unique setwhich is an extension of CBFS[11] referred to be as Cubic Bipolar Intuitionistic Fuzzy Sets (CBIFS)[12],. Here we nowintroduce the concept of cubic bipolar intuitionistic fuzzy prioritised weighted averaging aggregation operators (CBIFPWA), cubic bipolar intuitionistic fuzzy prioritised weighted geometric aggregation operators (CBIFPWG), cubic bipolar intuitionistic fuzzy prioritised averaging aggregation operators with priority degrees (CBIFPDA)" and "cubic bipolar intuitionistic fuzzy prioritised geometric aggregation operators with priority degrees (CBIFPDG)" for resolving issues with multi-criteria decision making (MCDM). The fundamental definitions that are not shown here can be found in [1] to [11].

**Definition 1.1.** [12] Let $I[0,1]$ be the set of all closed subintervals of[0,1]and $I^*[-1,0]$ be the set of all closed sub-intervals of$[-1,0]$. A cubic bipolar intuitionistic fuzzy set (CBIFS) $\tilde{\mathcal{C}}^I$ on $\mathcal{X}$ can be described as$\tilde{\mathcal{C}}^I = \{(x, \widetilde{\mathcal{B}}^I(x), \mathcal{B}^I(x))\}: x \in \mathcal{X}\}$where $\widetilde{\mathcal{B}}^I(x) = \{[\mu^{+l}_{\mathcal{B}^I}(x), \mu^{+u}_{\mathcal{B}^I}(x)], [\mu^{-l}_{\mathcal{B}^I}(x), \mu^{-l}_{\mathcal{B}^I}(x)], [\vartheta^{+l}_{\mathcal{B}^I}(x), \vartheta^{+u}_{\mathcal{B}^I}(x)], [\vartheta^{-l}_{\mathcal{B}^I}(x), \vartheta^{-u}_{\mathcal{B}^I}(x)]\}$ be an interval-valued bipolar intuitionistic fuzzy set (IVBIFS) and $\mathcal{B}^I(x) = \{\mu^+_{\mathcal{B}^I}(x), \mu^-_{\mathcal{B}^I}(x), \vartheta^+_{\mathcal{B}^I}(x), \vartheta^-_{\mathcal{B}^I}(x)\}$ be a bipolar intuitionistic fuzzy set (BIFS). A cubic bipolar intuitionistic fuzzy element (CBIFE) can be symbolized by $\tilde{\mathcal{C}}^I = \{\{[\mu^{+l}_{\mathcal{B}^I}, \mu^{+u}_{\mathcal{B}^I}], [\mu^{-l}_{\mathcal{B}^I}, \mu^{-l}_{\mathcal{B}^I}], [\vartheta^{+l}_{\mathcal{B}^I}, \vartheta^{+u}_{\mathcal{B}^I}], [\vartheta^{-l}_{\mathcal{B}^I}, \vartheta^{-u}_{\mathcal{B}^I}]\}, \{\mu^+_{\mathcal{B}^I}, \mu^-_{\mathcal{B}^I}, \vartheta^+_{\mathcal{B}^I}, \vartheta^-_{\mathcal{B}^I}\}\}$.

**Definition 1.2.**[12] Consider $\tilde{\mathcal{C}}^I_k = \{\{[\mu^{+l}_{\mathcal{B}^I k}, \mu^{+u}_{\mathcal{B}^I k}], [\mu^{-l}_{\mathcal{B}^I k}, \mu^{-u}_{\mathcal{B}^I k}], [\vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^{+u}_{\mathcal{B}^I k}], [\vartheta^{-l}_{\mathcal{B}^I k}, \vartheta^{-u}_{\mathcal{B}^I k}]\}, \{\mu^+_{\mathcal{B}^I k}, \mu^-_{\mathcal{B}^I k}, \vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^-_{\mathcal{B}^I k}\}\}, (k = 1,2, \dots, n)$to be the collection of CBIFEs and let the weighted vector be $\mathcal{W} = \{\mathfrak{w}_1, \mathfrak{w}_2, \dots \mathfrak{w}_n\}^T$ such that $\sum_{k=1}^n \mathfrak{w}_k = 1$and $0 \le \mathfrak{w}_k \le 1$. The $CBIFWG$ operator by the mapping $\mathcal{F} : \tilde{\mathcal{C}}^I_n \to \tilde{\mathcal{C}}^I$ as:

$$CBIFWG(\tilde{\mathcal{C}}^I_1, \tilde{\mathcal{C}}^I_2, \dots, \tilde{\mathcal{C}}^I_n) = \{\{[\prod_{k=1}^n (\mu^{+l}_{\mathcal{B}^I k})^{\mathfrak{w}_k}, \prod_{k=1}^n (\mu^{+u}_{\mathcal{B}^I k})^{\mathfrak{w}_k}], [-(1 - \prod_{k=1}^n (1 - (-\mu^{-l}_{\mathcal{B}^I k}))^{\mathfrak{w}_k}), -(1 - \prod_{k=1}^n (1 - (-\mu^{-u}_{\mathcal{B}^I k}))^{\mathfrak{w}_k})], [1 - \prod_{k=1}^n (1 - \vartheta^{+l}_{\mathcal{B}^I k})^{\mathfrak{w}_k}, 1 - \prod_{k=1}^n (1 - \vartheta^{+u}_{\mathcal{B}^I k})^{\mathfrak{w}_k}], [-\prod_{k=1}^n (-\vartheta^{-l}_{\mathcal{B}^I k})^{\mathfrak{w}_k}, -\prod_{k=1}^n (-\vartheta^{-u}_{\mathcal{B}^I k})^{\mathfrak{w}_k}]\}, \{\prod_{k=1}^n (\mu^+_{\mathcal{B}^I k})^{\mathfrak{w}_k}, -(1 - \prod_{k=1}^n (1 - (-\mu^-_{\mathcal{B}^I k}))^{\mathfrak{w}_k}), 1 - \prod_{k=1}^n (1 - \vartheta^+_{\mathcal{B}^I k})^{\mathfrak{w}_k}, -\prod_{k=1}^n (-\vartheta^-_{\mathcal{B}^I k})^{\mathfrak{w}_k}\}\} \quad (1)$$

**Definition 1.3.** [12] The Score function for CBIFEs can be computed as,

$$S(\tilde{\mathcal{C}}^I) = \frac{1}{12}\left\{\begin{array}{l} 4 + [\mu^{+l}_{\mathcal{B}^I k}(x) + \mu^{+u}_{\mathcal{B}^I k}(x)] + [\mu^{-l}_{\mathcal{B}^I k}(x) + \mu^{-u}_{\mathcal{B}^I k}(x)] - [\vartheta^{+l}_{\mathcal{B}^I k}(x) + \vartheta^{+u}_{\mathcal{B}^I k}(x)] \\ -[\vartheta^{-l}_{\mathcal{B}^I k}(x) + \vartheta^{-u}_{\mathcal{B}^I k}(x)] + 2 + \mu^+_{\mathcal{B}^I k}(x) + \mu^-_{\mathcal{B}^I k}(x) - \vartheta^+_{\mathcal{B}^I k}(x) - \vartheta^-_{\mathcal{B}^I k}(x) \end{array}\right\}$$

(2)

where $S(\tilde{\mathcal{C}}^I) \in [0,1]$. If $S(\tilde{\mathcal{C}}^I_1) < S(\tilde{\mathcal{C}}^I_2)$, then $\tilde{\mathcal{C}}^I_1 < \tilde{\mathcal{C}}^I_2$ and if $S(\tilde{\mathcal{C}}^I_1) > S(\tilde{\mathcal{C}}^I_2)$, then $\tilde{\mathcal{C}}^I_1 > \tilde{\mathcal{C}}^I_2$.

**Definition 1.4.** [12] The Accuracy function for CBIFEs can be computed as,

$$A(\tilde{\mathcal{C}}^I) = \frac{1}{12}\left\{\begin{array}{l} 4 + [\mu^{+l}_{\mathcal{B}^I k}(x) + \mu^{+u}_{\mathcal{B}^I k}(x)] - [\mu^{-l}_{\mathcal{B}^I k}(x) + \mu^{-u}_{\mathcal{B}^I k}(x)] - [\vartheta^{+l}_{\mathcal{B}^I k}(x) + \vartheta^{+u}_{\mathcal{B}^I k}(x)] \\ +[\vartheta^{-l}_{\mathcal{B}^I k}(x) + \vartheta^{-u}_{\mathcal{B}^I k}(x)] + 2 + \mu^+_{\mathcal{B}^I k}(x) - \mu^-_{\mathcal{B}^I k}(x) - \vartheta^+_{\mathcal{B}^I k}(x) + \vartheta^-_{\mathcal{B}^I k}(x) \end{array}\right\} \quad (3)$$

where $A(\tilde{\mathcal{C}}^I) \in [0,1]$. If$A(\tilde{\mathcal{C}}^I_1) > A(\tilde{\mathcal{C}}^I_2)$, then $\tilde{\mathcal{C}}^I_1 > \tilde{\mathcal{C}}^I_2$ and if $A(\tilde{\mathcal{C}}^I_1) = A(\tilde{\mathcal{C}}^I_2)$, then $\tilde{\mathcal{C}}^I_1 = \tilde{\mathcal{C}}^I_2$.

## CUBIC BIPOLAR INTUITIONISTIC FUZZY AVERAGING AGGREGATION OPERATORS WITH PRIORITY DEGREES :

Here, in this following section, we introduce thecubic bipolar intuitionistic fuzzy prioritized weighted averaging aggregation operators (AOs) and cubic bipolar intuitionistic fuzzy averaging AOs with priority degrees.

**Definition 2.1.**Consider $\tilde{\mathcal{C}}^I_k = \{\{[\mu^{+l}_{\mathcal{B}^I k}, \mu^{+u}_{\mathcal{B}^I k}], [\mu^{-l}_{\mathcal{B}^I k}, \mu^{-u}_{\mathcal{B}^I k}], [\vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^{+u}_{\mathcal{B}^I k}], [\vartheta^{-l}_{\mathcal{B}^I k}, \vartheta^{-u}_{\mathcal{B}^I k}]\}, \{\mu^+_{\mathcal{B}^I k}, \mu^-_{\mathcal{B}^I k}, \vartheta^+_{\mathcal{B}^I k}, \vartheta^-_{\mathcal{B}^I k}\}\}(k = 1,2, \dots, n)$to be the collection of CBIFEs. Then we defined the $CBIFPWA$ operator by the mapping $\mathcal{F} : \tilde{\mathcal{C}}^I_n \to \tilde{\mathcal{C}}^I$ as:

$CBIFPWA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \mathcal{r}_1\tilde{\mathcal{C}}_1^I \oplus \mathcal{r}_2\tilde{\mathcal{C}}_2^I \oplus \dots \oplus \mathcal{r}_n\tilde{\mathcal{C}}_n^I$ (4) where $\mathcal{r}_i = \frac{t_i}{\sum_{k=1}^n t_k}$ and $t_j = \prod_{k=1}^{j-1} S(\tilde{\mathcal{C}}_k^I)$ , $j = 2,3, \dots, n$, where $S(\tilde{\mathcal{C}}_k^I)$ be the score function of the $k^{th}$ $CBIFN$ and $t_1 = 1$.

**Definition 2.2.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\} (k = 1,2, \dots, n)$ to be the collection of CBIFEs. Then we defined the $CBIFPDA$ operator by the mapping $\mathcal{F} : \tilde{\mathcal{C}}_n^I \to \tilde{\mathcal{C}}^I$ as:

$CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \mathcal{r}_1^{(d)}\tilde{\mathcal{C}}_1^I \oplus \mathcal{r}_2^{(d)}\tilde{\mathcal{C}}_2^I \oplus \dots \oplus \mathcal{r}_n^{(d)}\tilde{\mathcal{C}}_n^I$

(5) where $\mathcal{r}_i^{(d)} = \frac{t_i^{(d)}}{\sum_{k=1}^n t_k^{(d)}}$ and $t_j^{(d)} = \prod_{k=1}^{j-1} (S(\tilde{\mathcal{C}}_k^I))^{d_k}$ , $j = 2,3, \dots, n$, where $S(\tilde{\mathcal{C}}_k^I)$ be the score function of the $k^{th}$ $CBIFN$ and $t_1 = 1$.

**Theorem 2.3.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\} (k = 1,2, \dots, n)$ to be the collection of CBIFEs. Then we defined the $CBIFPDA$ operator by the mapping $\mathcal{F} : \tilde{\mathcal{C}}_n^I \to \tilde{\mathcal{C}}^I$ as:

$$CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \mathcal{r}_1^{(d)}\tilde{\mathcal{C}}_1^I \oplus \mathcal{r}_2^{(d)}\tilde{\mathcal{C}}_2^I \oplus \dots \oplus \mathcal{r}_n^{(d)}\tilde{\mathcal{C}}_n^I$$

$$= \{\{[1 - \prod_{k=1}^{n} (1 - \mu_{\mathcal{B}^I k}^{+l})^{\mathcal{r}_k^{(d)}}, 1 - \prod_{k=1}^{n} (1 - \mu_{\mathcal{B}^I k}^{+u})^{\mathcal{r}_k^{(d)}}], [- \prod_{k=1}^{n} (-\mu_{\mathcal{B}^I k}^{-l})^{\mathcal{r}_k^{(d)}}, - \prod_{k=1}^{n} (-\mu_{\mathcal{B}^I k}^{-u})^{\mathcal{r}_k^{(d)}}], [\prod_{k=1}^{n} (\vartheta_{\mathcal{B}^I k}^{+l})^{\mathcal{r}_k^{(d)}}, \prod_{k=1}^{n} (\vartheta_{\mathcal{B}^I k}^{+u})^{\mathcal{r}_k^{(d)}}], [-(1 - \prod_{k=1}^{n} (1 - (-\vartheta_{\mathcal{B}^I k}^{-l}))^{\mathcal{r}_k^{(d)}}), -(1 - \prod_{k=1}^{n} (1 - (-\vartheta_{\mathcal{B}^I k}^{-u}))^{\mathcal{r}_k^{(d)}})]\}, \{ 1 - \prod_{k=1}^{n} (1 - \mu_{\mathcal{B}^I k}^{+})^{\mathcal{r}_k^{(d)}}, \{- \prod_{k=1}^{n} (-\mu_{\mathcal{B}^I k}^{-})^{\mathcal{r}_k^{(d)}}, \prod_{k=1}^{n} (\vartheta_{\mathcal{B}^I k}^{+})^{\mathcal{r}_k^{(d)}}, -(1 - \prod_{k=1}^{n} (1 - (-\vartheta_{\mathcal{B}^I k}^{-}))^{\mathcal{r}_k^{(d)}})\}\} \quad (6)$$

**Proof.** We demonstrate the aforementioned theorem by mathematical induction.
Let $n = 2$,

$$\mathcal{r}_1^{(d)}\tilde{\mathcal{C}}_1^I = \{\{[1 - (1 - \mu_{\mathcal{B}^I 1}^{+l})^{\mathcal{r}_1^{(d)}}, 1 - (1 - \mu_{\mathcal{B}^I 1}^{+u})^{\mathcal{r}_1^{(d)}}], [-(-\mu_{\mathcal{B}^I 1}^{-l})^{\mathcal{r}_1^{(d)}}, -(-\mu_{\mathcal{B}^I 1}^{-u})^{\mathcal{r}_1^{(d)}}], [(\vartheta_{\mathcal{B}^I 1}^{+l})^{\mathcal{r}_1^{(d)}}, (\vartheta_{\mathcal{B}^I 1}^{+u})^{\mathcal{r}_1^{(d)}}], [-(1 - (1 - (-\vartheta_{\mathcal{B}^I 1}^{-l}))^{\mathcal{r}_1^{(d)}}), -(1 - (1 - (-\vartheta_{\mathcal{B}^I 1}^{-u}))^{\mathcal{r}_1^{(d)}})]\}, \{1 - (1 - \mu_{\mathcal{B}^I 1}^{+})^{\mathcal{r}_1^{d_1}}, -(-\mu_{\mathcal{B}^I 1}^{-})^{\mathcal{r}_1^{(d)}}, (\vartheta_{\mathcal{B}^I 1}^{+})^{\mathcal{r}_1^{(d)}}, -(1 - (1 - (-\vartheta_{\mathcal{B}^I 1}^{-}))^{\mathcal{r}_1^{(d)}})\}\}$$

$$\mathcal{r}_2^{(d)}\tilde{\mathcal{C}}_2^I = \{\{[1 - (1 - \mu_{\mathcal{B}^I 2}^{+l})^{\mathcal{r}_2^{(d)}}, 1 - (1 - \mu_{\mathcal{B}^I 2}^{+u})^{\mathcal{r}_2^{(d)}}], [-(-\mu_{\mathcal{B}^I 2}^{-l})^{\mathcal{r}_2^{(d)}}, -(-\mu_{\mathcal{B}^I 2}^{-u})^{\mathcal{r}_2^{(d)}}], [(\vartheta_{\mathcal{B}^I 2}^{+l})^{\mathcal{r}_2^{(d)}}, (\vartheta_{\mathcal{B}^I 2}^{+u})^{\mathcal{r}_2^{(d)}}], [-(1 - (1 - (-\vartheta_{\mathcal{B}^I 2}^{-l}))^{\mathcal{r}_2^{(d)}}), -(1 - (1 - (-\vartheta_{\mathcal{B}^I 2}^{-u}))^{\mathcal{r}_2^{(d)}})]\}, \{1 - (1 - \mu_{\mathcal{B}^I 2}^{+})^{\mathcal{r}_2^{(d)}}, -(-\mu_{\mathcal{B}^I 2}^{-})^{\mathcal{r}_2^{(d)}}, (\vartheta_{\mathcal{B}^I 2}^{+})^{\mathcal{r}_2^{(d)}}, -(1 - (1 - (-\vartheta_{\mathcal{B}^I 2}^{-}))^{\mathcal{r}_2^{(d)}}\}\}$$

$$r_1^{(d)}\tilde{C}_1^I \oplus r_2^{(d)}\tilde{C}_2^I = \{\{[1-((1-\mu_{\mathcal{B}^{I_1}}^{+l})^{r_1^{(d)}}(1-\mu_{\mathcal{B}^{I_2}}^{+l})^{r_2^{(d)}}), 1 - ((1-\mu_{\mathcal{B}^{I_1}}^{+u})^{r_1^{(d)}}(1 - \mu_{\mathcal{B}^{I_2}}^{+u})^{r_2^{(d)}})], [-((-\mu_{\mathcal{B}^{I_1}}^{-l})^{r_1^{(d)}}(-\mu_{\mathcal{B}^{I_2}}^{-l})^{r_2^{(d)}}), -((-\mu_{\mathcal{B}^{I_1}}^{-u})^{r_1^{(d)}}(-\mu_{\mathcal{B}^{I_2}}^{-u})^{r_2^{(d)}})], [((\vartheta_{\mathcal{B}^{I_1}}^{+l})^{r_1^{(d)}}(\vartheta_{\mathcal{B}^{I_2}}^{+l})^{r_2^{(d)}}), ((\vartheta_{\mathcal{B}^{I_1}}^{+u})^{r_1^{(d)}}(\vartheta_{\mathcal{B}^{I_2}}^{+u})^{r_2^{(d)}})], [-(1-(1-(-\vartheta_{\mathcal{B}^{I_1}}^{-l}))^{r_1^{(d)}}(1-(-\vartheta_{\mathcal{B}^{I_2}}^{-l}))^{r_2^{(d)}}), -(1 - (1-(-\vartheta_{\mathcal{B}^{I_1}}^{-u}))^{r_1^{(d)}}(1-(-\vartheta_{\mathcal{B}^{I_2}}^{-u}))^{r_2^{(d)}})]\}, \{1 - ((1-\mu_{\mathcal{B}^{I_1}}^{+})^{r_1^{(d)}}(1-\mu_{\mathcal{B}^{I_2}}^{+})^{r_2^{(d)}}), -((-\mu_{\mathcal{B}^{I_1}}^{-})^{r_1^{(d)}}(-\mu_{\mathcal{B}^{I_2}}^{-})^{r_2^{(d)}}), ((\vartheta_{\mathcal{B}^{I_1}}^{+})^{r_1^{(d)}}(\vartheta_{\mathcal{B}^{I_2}}^{+})^{r_2^{(d)}}), -(1 - (1-(-\vartheta_{\mathcal{B}^{I_1}}^{-}))^{r_1^{(d)}}(1-(-\vartheta_{\mathcal{B}^{I_2}}^{-}))^{r_2^{(d)}})\}\}$$

$$r_1^{(d)}\tilde{C}_1^I \oplus r_2^{(d)}\tilde{C}_2^I = \{\{[1-\prod_{k=1}^{2}(1-\mu_{\mathcal{B}^{I_k}}^{+l})^{r_k^{(d)}}, 1-\prod_{k=1}^{2}(1-\mu_{\mathcal{B}^{I_k}}^{+u})^{r_k^{(d)}}], [-\prod_{k=1}^{2}(-\mu_{\mathcal{B}^{I_k}}^{-l})^{r_k^{(d)}}, -\prod_{k=1}^{2}(-\mu_{\mathcal{B}^{I_k}}^{-u})^{r_k^{(d)}}], [\prod_{k=1}^{2}(\vartheta_{\mathcal{B}^{I_k}}^{+l})^{r_k^{(d)}}, \prod_{k=1}^{2}(\vartheta_{\mathcal{B}^{I_k}}^{+u})^{r_k^{(d)}}], [-(1-\prod_{k=1}^{2}(1-(-\vartheta_{\mathcal{B}^{I_k}}^{-l}))^{r_k^{(d)}}), -\left(1-\prod_{k=1}^{2}(1-(-\vartheta_{\mathcal{B}^{I_k}}^{-u}))^{r_k^{(d)}}\right)], \{1-\prod_{k=1}^{2}(1-\mu_{\mathcal{B}^{I_k}}^{+})^{r_k^{(d)}}, -\prod_{k=1}^{2}(-\mu_{\mathcal{B}^{I_k}}^{-})^{r_k^{(d)}}, \prod_{k=1}^{2}(\vartheta_{\mathcal{B}^{I_k}}^{+})^{r_k^{(d)}}, -(1-\prod_{k=1}^{2}(1-(-\vartheta_{\mathcal{B}^{I_k}}^{-}))^{r_k^{(d)}})\}\}$$

It holds for$n = 2$.

Assume Equation. (6) holds true for $n = r$, (i.e.)

$$CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_r^I) = r_1^{(d)}\tilde{C}_1^I \oplus r_2^{(d)}\tilde{C}_2^I \oplus \dots \oplus r_r^{(d)}\tilde{C}_r^I$$

$$= \{\{[1-\prod_{k=1}^{r}(1-\mu_{\mathcal{B}^{I_k}}^{+l})^{r_k^{(d)}}, 1-\prod_{k=1}^{r}(1-\mu_{\mathcal{B}^{I_k}}^{+u})^{r_k^{(d)}}], [-\prod_{k=1}^{r}(-\mu_{\mathcal{B}^{I_k}}^{-l})^{r_k^{(d)}}, -\prod_{k=1}^{r}(-\mu_{\mathcal{B}^{I_k}}^{-u})^{r_k^{(d)}}], [\prod_{k=1}^{r}(\vartheta_{\mathcal{B}^{I_k}}^{+l})^{r_k^{(d)}}, \prod_{k=1}^{r}(\vartheta_{\mathcal{B}^{I_k}}^{+u})^{r_k^{(d)}}], [-(1-$$

$$\prod_{k=1}^{r}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-l})\right)^{\mathcal{r}_k{}^{(d)}}), -(1-\prod_{k=1}^{r}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-u})\right)^{\mathcal{r}_k{}^{(d)}})]\}, \{1 - \prod_{k=1}^{r}\left(1 - \mu_{\mathcal{B}^{I}k}^{+}\right)^{\mathcal{r}_k{}^{(d)}}, \{-\prod_{k=1}^{r}\left(-\mu_{\mathcal{B}^{I}k}^{-}\right)^{\mathcal{r}_k{}^{(d)}}, \prod_{k=1}^{r}\left(\vartheta_{\mathcal{B}^{I}k}^{+}\right)^{\mathcal{r}_k{}^{(d)}}, -(1-\prod_{k=1}^{r}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-})\right)^{\mathcal{r}_k{}^{(d)}})\}\}$$

We now need to demonstrate that the equation.(6) is valid for $n = r + 1$.

$$CBIFPDA\left(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \ldots, \tilde{\mathcal{C}}_{r+1}^I\right) = \mathcal{r}_1{}^{(d)}\tilde{\mathcal{C}}_1^I \oplus \mathcal{r}_2{}^{(d)}\tilde{\mathcal{C}}_2^I \oplus \ldots \oplus \mathcal{r}_r{}^{(d)}\tilde{\mathcal{C}}_r^I \oplus \mathcal{r}_{r+1}{}^{(d)}\tilde{\mathcal{C}}_{r+1}^I$$

$$= \{\{[1-\prod_{k=1}^{r}\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k{}^{(d)}}, 1-\prod_{k=1}^{r}\left(1-\mu_{\mathcal{B}^{I}k}^{+u}\right)^{\mathcal{r}_k{}^{(d)}}], [-\prod_{k=1}^{r}\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k{}^{(d)}}, -\prod_{k=1}^{r}\left(-\mu_{\mathcal{B}^{I}k}^{-u}\right)^{\mathcal{r}_k{}^{(d)}}], [\prod_{k=1}^{r}\left(\vartheta_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k{}^{(d)}}, \prod_{k=1}^{r}\left(\vartheta_{\mathcal{B}^{I}k}^{+u}\right)^{\mathcal{r}_k{}^{(d)}}],$$

$$[-(1-\prod_{k=1}^{r}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-l})\right)^{\mathcal{r}_k{}^{(d)}}), -\left(1-\prod_{k=1}^{r}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-u})\right)^{\mathcal{r}_k{}^{(d)}}\right)], \{1 - \prod_{k=1}^{r}\left(1 - \mu_{\mathcal{B}^{I}k}^{+}\right)^{\mathcal{r}_k{}^{(d)}}, -\prod_{k=1}^{r}\left(-\mu_{\mathcal{B}^{I}k}^{-}\right)^{\mathcal{r}_k{}^{(d)}}, \prod_{k=1}^{r}\left(\vartheta_{\mathcal{B}^{I}k}^{+}\right)^{\mathcal{r}_k{}^{(d)}}, -(1 - \prod_{k=1}^{r}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-})\right)^{\mathcal{r}_k{}^{(d)}})\}\} \oplus$$

$$\{\{[1-\left(1-\mu_{\mathcal{B}^{I}r+1}^{+l}\right)^{\mathcal{r}_{r+1}{}^{(d)}}, 1-\left(1-\mu_{\mathcal{B}^{I}r+1}^{+u}\right)^{\mathcal{r}_{r+1}{}^{(d)}}], [-\left(-\mu_{\mathcal{B}^{I}r+1}^{-l}\right)^{\mathcal{r}_{r+1}{}^{(d)}}, -\left(-\mu_{\mathcal{B}^{I}r+1}^{-u}\right)^{\mathcal{r}_{r+1}{}^{(d)}}], [\left(\vartheta_{\mathcal{B}^{I}r+1}^{+l}\right)^{\mathcal{r}_{r+1}{}^{(d)}}, \left(\vartheta_{\mathcal{B}^{I}r+1}^{+u}\right)^{\mathcal{r}_{r+1}{}^{(d)}}], [-(1-(1-\left(\vartheta_{\mathcal{B}^{I}r+1}^{-l}\right))^{\mathcal{r}_{r+1}{}^{(d)}}), -(1 - \left(1-(-\vartheta_{\mathcal{B}^{I}r+1}^{-u})\right)^{\mathcal{r}_{r+1}{}^{(d)}})]\}, \{1 - \left(1 - \mu_{\mathcal{B}^{I}r+1}^{+}\right)^{\mathcal{r}_{r+1}{}^{(d)}}, -\left(-\mu_{\mathcal{B}^{I}r+1}^{-}\right)^{\mathcal{r}_{r+1}{}^{(d)}}, \left(\vartheta_{\mathcal{B}^{I}r+1}^{+l}\right)^{\mathcal{r}_{r+1}{}^{(d)}}, -(1 - \left(1-(-\vartheta_{\mathcal{B}^{I}r+1}^{-})\right)^{\mathcal{r}_{r+1}{}^{(d)}}\}\}$$

$$= \{\{[1-\prod_{k=1}^{r+1}\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k{}^{(d)}}, 1-\prod_{k=1}^{r+1}\left(1-\mu_{\mathcal{B}^{I}k}^{+u}\right)^{\mathcal{r}_k{}^{(d)}}], [-\prod_{k=1}^{r+1}\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k{}^{(d)}}, -\prod_{k=1}^{r+1}\left(-\mu_{\mathcal{B}^{I}k}^{-u}\right)^{\mathcal{r}_k{}^{(d)}}], [\prod_{k=1}^{r+1}\left(\vartheta_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k{}^{(d)}},$$

$$\prod_{k=1}^{r+1}\left(\vartheta_{\mathcal{B}^I k}^{+u}\right)^{r_k^{(d)}}],[-(1-\prod_{k=1}^{r+1}\left(1-(-\vartheta_{\mathcal{B}^I k}^{-l})\right)^{r_k^{(d)}}),-(1-\prod_{k=1}^{r+1}\left(1-(-\vartheta_{\mathcal{B}^I k}^{-u})\right)^{r_k^{(d)}})]\},\{1-\prod_{k=1}^{r+1}\left(1-\mu_{\mathcal{B}^I k}^{+}\right)^{r_k^{(d)}},-\prod_{k=1}^{r+1}\left(-\mu_{\mathcal{B}^I k}^{-}\right)^{r_k^{(d)}},\prod_{k=1}^{r+1}\left(\vartheta_{\mathcal{B}^I k}^{+}\right)^{r_k^{(d)}},-(1-\prod_{k=1}^{r+1}\left(1-(-\vartheta_{\mathcal{B}^I k}^{-})\right)^{r_k^{(d)}})\}\}$$

The above proves that eqn. (6) holds for $n = r + 1$. Then,

$$CBIFPDA\left(\tilde{C}_1^I,\tilde{C}_2^I,\dots,\tilde{C}_n^I\right) = \{\{[1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^I k}^{+l}\right)^{r_k^{(d)}},1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^I k}^{+u}\right)^{r_k^{(d)}}],[-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^I k}^{-l}\right)^{r_k^{(d)}},-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^I k}^{-u}\right)^{r_k^{(d)}}],[\prod_{k=1}^{n}\left(\vartheta_{\mathcal{B}^I k}^{+l}\right)^{r_k^{(d)}},\prod_{k=1}^{n}\left(\vartheta_{\mathcal{B}^I k}^{+u}\right)^{r_k^{(d)}}],[-(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^I k}^{-l})\right)^{r_k^{(d)}}),-(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^I k}^{-u})\right)^{r_k^{(d)}})]\},\{1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^I k}^{+}\right)^{r_k^{(d)}},\{-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^I k}^{-}\right)^{r_k^{(d)}},\prod_{k=1}^{n}\left(\vartheta_{\mathcal{B}^I k}^{+}\right)^{r_k^{(d)}},-(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^I k}^{-})\right)^{r_k^{(d)}})\}\}\square$$

**Example 2.4.** Let $\tilde{C}_1^I,\tilde{C}_2^I,\tilde{C}_3^I$ *and* $\tilde{C}_4^I$ be the four CBIFEs as:

$\tilde{C}_1^I = \{\{[0.23,0.43],[-0.33,-0.23],[0.33,0.47],[-0.52,-0.41]\},\{0.35,-0.29,0.40,-0.47\}\}$
$\tilde{C}_2^I = \{\{[0.17,0.28],[-0.63,-0.56],[0.46,0.51],[-0.26,-0.19]\},\{0.20,-0.60,0.50,-0.23\}\}$
$\tilde{C}_3^I = \{\{[0.37,0.45],[-0.71,-0.55],[0.50,0.52],[-0.23,-0.16]\},\{0.41,-0.58,0.51,-0.21\}\}$
$\tilde{C}_4^I = \{\{[0.50,0.60],[-0.40,-0.30],[0.31,0.39],[-0.50,-0.40]\},\{0.55,-0.35,0.36,-0.45\}\}$

Calculate the CBIF AO and consider the priority degree as $d = (2,1,1)$.

**Solution:** Now, we compute the score for each CBIFEs:

$S(\tilde{C}_1^I) = 0.5300, S(\tilde{C}_2^I) = 0.3392, S(\tilde{C}_3^I) = 0.3717, S(\tilde{C}_4^I) = 0.5742$
$t_1 = 1, t_2 = 0.5300, t_3 = 0.1798, t_4 = 0.0668$
$r_1^{(d)} = 0.5629, r_2^{(d)} = 0.2983, r_3^{(d)} = 0.1012, r_4^{(d)} = 0.0376$

By using eqn. (5), we get

$$CBIFPDA\left(\tilde{C}_1^I,\tilde{C}_2^I,\tilde{C}_3^I,\tilde{C}_4^I\right) = \{\{[0.24,0.40],[-0.44,-0.33],[0.38,0.48],[-0.43,-0.33]\},\{0.32,-0.39,0.44,-0.38\}\}$$

**Theorem 2.5.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l},\mu_{\mathcal{B}^I k}^{+u}],[\mu_{\mathcal{B}^I k}^{-l},\mu_{\mathcal{B}^I k}^{-u}],[\vartheta_{\mathcal{B}^I k}^{+l},\vartheta_{\mathcal{B}^I k}^{+u}],[\vartheta_{\mathcal{B}^I k}^{-l},\vartheta_{\mathcal{B}^I k}^{-u}]\},\{\mu_{\mathcal{B}^I k}^{+},\mu_{\mathcal{B}^I k}^{-},\vartheta_{\mathcal{B}^I k}^{+},\vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\dots,n)$to be the collection of CBIFEs. Then, the $CBIFPDA$ operator satisfies the idempotency property (i.e) if $\tilde{\mathcal{C}}_k^I = \tilde{\mathcal{C}}^I, \forall k = 1,2,\dots,n$ we have $CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \tilde{\mathcal{C}}^I$.

**Proof.** By considering eqn. (5), we have

$$CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = r_1^{(d)}\tilde{\mathcal{C}}_1^I \oplus r_2^{(d)}\tilde{\mathcal{C}}_2^I \oplus \dots \oplus r_n^{(d)}\tilde{\mathcal{C}}_n^I$$

$$= \frac{t_1^{(d)}}{\sum_{k=1}^n t_k^{(d)}}\tilde{\mathcal{C}}_1^I \oplus \frac{t_2^{(d)}}{\sum_{k=1}^n t_k^{(d)}}\tilde{\mathcal{C}}_2^I \oplus \dots \oplus \frac{t_n^{(d)}}{\sum_{k=1}^n t_k^{(d)}}\tilde{\mathcal{C}}_n^I$$

$$= \frac{t_1^{(d)}}{\sum_{k=1}^n t_k^{(d)}}\tilde{\mathcal{C}}^I \oplus \frac{t_2^{(d)}}{\sum_{k=1}^n t_k^{(d)}}\tilde{\mathcal{C}}^I \oplus \dots \oplus \frac{t_n^{(d)}}{\sum_{k=1}^n t_k^{(d)}}\tilde{\mathcal{C}}^I (\because \tilde{\mathcal{C}}_k^I = \tilde{\mathcal{C}}^I,\ \forall k)$$

$$= \frac{\sum_{k=1}^n t_k^{(d)}}{\sum_{k=1}^n t_k^{(d)}}\tilde{\mathcal{C}}^I$$

$$= 1.\tilde{\mathcal{C}}^I$$

$$CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \tilde{\mathcal{C}}^I \square$$

**Theorem 2.6.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\dots,n)$ as the collection of CBIFEs. Then, the $CBIFPDA$ operator adheres the monotonicity property (i.e.) if $\tilde{\mathcal{C}}_k^I \le \tilde{\mathcal{C}}_k^{I'}, \forall k = 1,2,\dots,n$, we get $CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) \le CBIFPDA(\tilde{\mathcal{C}}_1^{I'}, \tilde{\mathcal{C}}_2^{I'}, \dots, \tilde{\mathcal{C}}_n^{I'})$.

**Proof.** If $\mu_{\mathcal{B}^I k}^{+l} \le \mu_{\mathcal{B}^{I'} k}^{+l}$, then $1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^I k}^{+l})^{r_k^{(d)}} \le 1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^{I'} k}^{+l})^{r_k^{(d)}}$

Similarly, if $\mu_{\mathcal{B}^I k}^{+u} \le \mu_{\mathcal{B}^{I'} k}^{+u}$, then $1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^I k}^{+u})^{r_k^{(d)}} \le 1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^{I'} k}^{+u})^{r_k^{(d)}}$

If $\mu_{\mathcal{B}^I k}^{-l} \le \mu_{\mathcal{B}^{I'} k}^{-l}$, then $-\prod_{k=1}^n (-\mu_{\mathcal{B}^I k}^{-l})^{r_k^{(d)}} \le -\prod_{k=1}^n (-\mu_{\mathcal{B}^{I'} k}^{-l})^{r_k^{(d)}}$

Similarly, if $\mu_{\mathcal{B}^I k}^{-u} \le \mu_{\mathcal{B}^{I'} k}^{-u}$, then $-\prod_{k=1}^n (-\mu_{\mathcal{B}^I k}^{-u})^{r_k^{(d)}} \le -\prod_{k=1}^n (-\mu_{\mathcal{B}^{I'} k}^{-u})^{r_k^{(d)}}$

Now, if $\vartheta_{\mathcal{B}^I k}^{+l} \le \vartheta_{\mathcal{B}^{I'} k}^{+l}$, then $\prod_{k=1}^n (\vartheta_{\mathcal{B}^I k}^{+l})^{r_k^{(d)}} \le \prod_{k=1}^n (\vartheta_{\mathcal{B}^{I'} k}^{+l})^{r_k^{(d)}}$

Similarly, if $\vartheta_{\mathcal{B}^I k}^{+u} \le \vartheta_{\mathcal{B}^{I'} k}^{+u}$, then $\prod_{k=1}^n (\vartheta_{\mathcal{B}^I k}^{+u})^{r_k^{(d)}} \le \prod_{k=1}^n (\vartheta_{\mathcal{B}^{I'} k}^{+u})^{r_k^{(d)}}$

Now, if $\vartheta_{\mathcal{B}^I k}^{-l} \le \vartheta_{\mathcal{B}^{I'} k}^{-l}$, then $-\left(1 - \prod_{k=1}^n (1 - (-\vartheta_{\mathcal{B}^I k}^{-l}))^{r_k^{(d)}}\right) \le -(1 - \prod_{k=1}^n \left(1 - (-\vartheta_{\mathcal{B}^{I'} k}^{-l})\right)^{r_k^{(d)}})$

Similarly, $\vartheta_{\mathcal{B}^I k}^{-u} \le \vartheta_{\mathcal{B}^{I'} k}^{-u}$, then $-\left(1 - \prod_{k=1}^n (1 - (-\vartheta_{\mathcal{B}^I k}^{-u}))^{r_k^{(d)}}\right) \le -(1 - \prod_{k=1}^n \left(1 - (-\vartheta_{\mathcal{B}^{I'} k}^{-u})\right)^{r_k^{(d)}})$

If $\mu_{\mathcal{B}^I k}^{+} \le \mu_{\mathcal{B}^{I'} k}^{+}$, then $1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^I k}^{+})^{r_k^{(d)}} \le 1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^{I'} k}^{+})^{r_k^{(d)}}$

If $\mu_{\mathcal{B}^I k}^{-} \le \mu_{\mathcal{B}^{I'} k}^{-}$, then $-\prod_{k=1}^n (-\mu_{\mathcal{B}^I k}^{-})^{r_k^{(d)}} \le -\prod_{k=1}^n (-\mu_{\mathcal{B}^{I'} k}^{-})^{r_k^{(d)}}$

If $\vartheta_{\mathcal{B}^I k}^{+} \le \vartheta_{\mathcal{B}^{I'} k}^{+}$, then $\prod_{k=1}^n (\vartheta_{\mathcal{B}^I k}^{+})^{r_k^{(d)}} \le \prod_{k=1}^n (\vartheta_{\mathcal{B}^{I'} k}^{+})^{r_k^{(d)}}$

If $\vartheta_{\mathcal{B}^I k}^{-} \le \vartheta_{\mathcal{B}^{I'} k}^{-}$, then $-\left(1 - \prod_{k=1}^n (1 - (-\vartheta_{\mathcal{B}^I k}^{-}))^{r_k^{(d)}}\right) \le -(1 - \prod_{k=1}^n \left(1 - (-\vartheta_{\mathcal{B}^{I'} k}^{-})\right)^{r_k^{(d)}})$

Finally, we get

$$CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I)$$

$$= \{\{[1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^I k}^{+l})^{r_k^{(d)}}, 1 - \prod_{k=1}^n (1 - \mu_{\mathcal{B}^I k}^{+u})^{r_k^{(d)}}], [-\prod_{k=1}^n (-\mu_{\mathcal{B}^I k}^{-l})^{r_k^{(d)}},$$

$$-\prod_{k=1}^n (-\mu_{\mathcal{B}^I k}^{-u})^{r_k^{(d)}}], [\prod_{k=1}^n (\vartheta_{\mathcal{B}^I k}^{+l})^{r_k^{(d)}}, \prod_{k=1}^n (\vartheta_{\mathcal{B}^I k}^{+u})^{r_k^{(d)}}], [-(1 -$$

$$\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-l})\right)^{\mathcal{r}_k^{(d)}}),-(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-u})\right)^{\mathcal{r}_k^{(d)}})]\},\{1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+}\right)^{\mathcal{r}_k^{(d)}},\{-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I}k}^{-}\right)^{\mathcal{r}_k^{(d)}},\prod_{k=1}^{n}\left(\vartheta_{\mathcal{B}^{I}k}^{+}\right)^{\mathcal{r}_k^{(d)}},-(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^{I}k}^{-})\right)^{\mathcal{r}_k^{(d)}})\}\}$$

$$\leq \{\{[1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I'}k}^{+l}\right)^{\mathcal{r}_k^{(d)}},1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I'}k}^{+u}\right)^{\mathcal{r}_k^{(d)}}],[-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I'}k}^{-l}\right)^{\mathcal{r}_k^{(d)}},-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I'}k}^{-u}\right)^{\mathcal{r}_k^{(d)}}],[\prod_{k=1}^{n}\left(\vartheta_{\mathcal{B}^{I'}k}^{+l}\right)^{\mathcal{r}_k^{(d)}},\prod_{k=1}^{n}\left(\vartheta_{\mathcal{B}^{I'}k}^{+u}\right)^{\mathcal{r}_k^{(d)}}],-(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^{I'}k}^{-l})\right)^{\mathcal{r}_k^{(d)}}),-\left(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^{I'}k}^{-u})\right)^{\mathcal{r}_k^{(d)}}\right)],\{1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I'}k}^{+}\right)^{\mathcal{r}_k^{(d)}},-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I'}k}^{-}\right)^{\mathcal{r}_k^{(d)}},\prod_{k=1}^{n}\left(\vartheta_{\mathcal{B}^{I'}k}^{+}\right)^{\mathcal{r}_k^{(d)}},-(1-\prod_{k=1}^{n}\left(1-(-\vartheta_{\mathcal{B}^{I'}k}^{-})\right)^{\mathcal{r}_k^{(d)}})\}\}$$

$$CBIFPDA\left(\tilde{\mathcal{C}}_1^{I},\tilde{\mathcal{C}}_2^{I},\dots,\tilde{\mathcal{C}}_n^{I}\right)\leq CBIFPDA\left(\tilde{\mathcal{C}}_1^{I'},\tilde{\mathcal{C}}_2^{I'},\dots,\tilde{\mathcal{C}}_n^{I'}\right)$$

From the above, we have proven that the $CBIFPDA$ operator satisfies the monotonicity property.□

**Theorem 2.7.**Consider $\tilde{\mathcal{C}}_k^{I}=\{\{[\mu_{\mathcal{B}^{I}k}^{+l},\mu_{\mathcal{B}^{I}k}^{+u}],[\mu_{\mathcal{B}^{I}k}^{-l},\mu_{\mathcal{B}^{I}k}^{-u}],[\vartheta_{\mathcal{B}^{I}k}^{+l},\vartheta_{\mathcal{B}^{I}k}^{+u}],[\vartheta_{\mathcal{B}^{I}k}^{-l},\vartheta_{\mathcal{B}^{I}k}^{-u}]\},\{\mu_{\mathcal{B}^{I}k}^{+},\mu_{\mathcal{B}^{I}k}^{-},\vartheta_{\mathcal{B}^{I}k}^{+l},\vartheta_{\mathcal{B}^{I}k}^{-}\}\}$ $(k=1,2,\dots,n)$to be the collection of CBIFEs. Then, the $CBIFPDA$ operator satisfies the boundedness property.

**Proof.** We know that $\min_k(\mu_{\mathcal{B}^{I}k}^{+l})\leq\mu_{\mathcal{B}^{I}k}^{+l}\leq\max_k(\mu_{\mathcal{B}^{I}k}^{+l})$

$$\prod_{k=1}^{n}\min_k\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k^{(d)}}\geq\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k^{(d)}}\geq\prod_{k=1}^{n}\max_k\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k^{(d)}}$$

Then, $\min_k(1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k^{(d)}})\leq 1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k^{(d)}}\leq\max_k(1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+l}\right)^{\mathcal{r}_k^{(d)}})$

Similarly, $\min_k(1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+u}\right)^{\mathcal{r}_k^{(d)}})\leq 1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+u}\right)^{\mathcal{r}_k^{(d)}}\leq\max_k(1-\prod_{k=1}^{n}\left(1-\mu_{\mathcal{B}^{I}k}^{+u}\right)^{\mathcal{r}_k^{(d)}})$

Now, $\min_k\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)\geq-\mu_{\mathcal{B}^{I}k}^{-l}\geq\max_k(-\mu_{\mathcal{B}^{I}k}^{-l})$

$$\prod_{k=1}^{n}\min_k\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k^{(d)}}\geq\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k^{(d)}}\geq\prod_{k=1}^{n}\max_k\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k^{(d)}}$$

Then, $\min_k(-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k^{(d)}})\leq-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k^{(d)}}\leq\max_k(-\prod_{k=1}^{n}\left(-\mu_{\mathcal{B}^{I}k}^{-l}\right)^{\mathcal{r}_k^{(d)}})$

Similarly, $\min_k(-\prod_{k=1}^{n}(-\mu_{\mathcal{B}^I k}^{-u})^{r_k^{(d)}}) \leq -\prod_{k=1}^{n}(-\mu_{\mathcal{B}^I k}^{-u})^{r_k^{(d)}} \leq \max_k(-\prod_{k=1}^{n}(-\mu_{\mathcal{B}^I k}^{-u})^{r_k^{(d)}})$

Now, $\min_k(\vartheta_{\mathcal{B}^I k}^{+l}) \leq \vartheta_{\mathcal{B}^I k}^{+l} \leq \max_k(\vartheta_{\mathcal{B}^I k}^{+l})$

Then $\min_k(\prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+l})^{r_k^{(d)}}) \leq \prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+l})^{r_k^{(d)}} \leq \max_k(\prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+l})^{r_k^{(d)}})$

Similarly, $\min_k(\prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+u})^{r_k^{(d)}}) \leq \prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+u})^{r_k^{(d)}} \leq \max_k(\prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+u})^{r_k^{(d)}})$

Here $\min_k(-\vartheta_{\mathcal{B}^I k}^{-l}) \geq -\vartheta_{\mathcal{B}^I k}^{-l} \geq \max_k(-\vartheta_{\mathcal{B}^I k}^{-l})$

$$\min_k\left(\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-l}))^{r_k^{(d)}}\right) \leq \prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-l}))^{r_k^{(d)}} \leq \max_k(\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-l}))^{r_k^{(d)}})$$

Then, $\min_k -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-l}))^{r_k^{(d)}}\right) \leq -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-l}))^{r_k^{(d)}}\right) \leq \max_k -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-l}))^{r_k^{(d)}}\right)$

Similarly, $\min_k -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-u}))^{r_k^{(d)}}\right) \leq -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-u}))^{r_k^{(d)}}\right) \leq \max_k -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-u}))^{r_k^{(d)}}\right)$

A similar computation says,

$$\min_k(1-\prod_{k=1}^{n}(1-\mu_{\mathcal{B}^I k}^{+})^{r_k^{(d)}}) \leq 1-\prod_{k=1}^{n}(1-\mu_{\mathcal{B}^I k}^{+})^{r_k^{(d)}} \leq \max_k(1-\prod_{k=1}^{n}(1-\mu_{\mathcal{B}^I k}^{+})^{r_k^{(d)}})$$

$$\min_k(-\prod_{k=1}^{n}(-\mu_{\mathcal{B}^I k}^{-})^{r_k^{(d)}}) \leq -\prod_{k=1}^{n}(-\mu_{\mathcal{B}^I k}^{-})^{r_k^{(d)}} \leq \max_k(-\prod_{k=1}^{n}(-\mu_{\mathcal{B}^I k}^{-})^{r_k^{(d)}})$$

$$\min_k(\prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+})^{r_k^{(d)}}) \leq \prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+})^{r_k^{(d)}} \leq \max_k(\prod_{k=1}^{n}(\vartheta_{\mathcal{B}^I k}^{+})^{r_k^{(d)}})$$

$$\min_k -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-}))^{r_k^{(d)}}\right) \leq -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-}))^{r_k^{(d)}}\right)$$
$$\leq \max_k -\left(1-\prod_{k=1}^{n}(1-(-\vartheta_{\mathcal{B}^I k}^{-}))^{r_k^{(d)}}\right)$$

From the above all inequalities, we conclude that $CBIFPDA$ operator satisfies the boundedness property.□

**Theorem 2.8.**Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}(k = 1,2,\dots,n)$ and $\tilde{C}_k^{I'} = \{\{[\mu_{\mathcal{B}^{I'_k}}^{+l}, \mu_{\mathcal{B}^{I'_k}}^{+u}], [\mu_{\mathcal{B}^{I'_k}}^{-l}, \mu_{\mathcal{B}^{I'_k}}^{-u}], [\vartheta_{\mathcal{B}^{I'_k}}^{+l}, \vartheta_{\mathcal{B}^{I'_k}}^{+u}], [\vartheta_{\mathcal{B}^{I'_k}}^{-l}, \vartheta_{\mathcal{B}^{I'_k}}^{-u}]\}, \{\mu_{\mathcal{B}^{I'_k}}^{+}, \mu_{\mathcal{B}^{I'_k}}^{-}, \vartheta_{\mathcal{B}^{I'_k}}^{+}, \vartheta_{\mathcal{B}^{I'_k}}^{-}\}\}$ $(k = 1,2,\dots,n)$be the two assemblage of CBIFEs and let $\varphi > 0$, then we have the following properties:

1. $CBIFPDA(\tilde{C}_1^I \oplus \tilde{C}_1^{I'}, \tilde{C}_2^I \oplus \tilde{C}_1^{I'}, \dots, \tilde{C}_n^I \oplus \tilde{C}_1^{I'}) = CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) \oplus \tilde{C}_1^{I'}$
2. $CBIFPDA(\varphi\tilde{C}_1^I, \varphi\tilde{C}_2^I, \dots, \varphi\tilde{C}_n^I) = \varphi\, CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I)$
3. $CBIFPDA(\varphi\tilde{C}_1^I \oplus \tilde{C}_1^{I'}, \varphi\tilde{C}_2^I \oplus \tilde{C}_1^{I'}, \dots, \varphi\tilde{C}_n^I \oplus \tilde{C}_1^{I'}) = \varphi CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) \oplus \tilde{C}_1^{I'}$
4. $CBIFPDA(\tilde{C}_1^I \oplus \tilde{C}_1^{I'}, \tilde{C}_2^I \oplus \tilde{C}_2^{I'}, \dots, \tilde{C}_n^I \oplus \tilde{C}_n^{I'}) = CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) \oplus CBIFPDA(\tilde{C}_1^{I'}, \tilde{C}_2^{I'}, \dots, \tilde{C}_n^{I'})$

**Proof.** Proofs are trivial by the Definition 2.2.□

**Property 2.9.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}(k = 1,2, \dots, n)$to be the collection of CBIFEs. Then, $\lim_{(d_1,d_2,\dots,d_{n-1})\to(1,1,\dots,1)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = CBIFPWA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I)$.

**Proof.** From given, $(d_1, d_2, \dots, d_{n-1}) \to (1,1, \dots,1)$, we get

$\mathrm{t}_k{}^{(d)} = \prod_{k=1}^{j-1}(S(\tilde{C}_k^I))^{d_k} \to \prod_{k=1}^{j-1}(S(\tilde{C}_k^I)) = \mathrm{t}_k$and$\mathscr{r}_i{}^{(d)} = \mathscr{r}_i$

Then,

$$\lim_{(d_1,d_2,\dots,d_{n-1})\to(1,1,\dots,1)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I)$$
$$= \lim_{(d_1,d_2,\dots,d_{n-1})\to(1,1,\dots,1)} \mathscr{r}_1{}^{(d)}\tilde{C}_1^I \oplus \mathscr{r}_2{}^{(d)}\tilde{C}_2^I \oplus \dots \oplus \mathscr{r}_n{}^{(d)}\tilde{C}_n^I$$
$$= \mathscr{r}_1\tilde{C}_1^I \oplus \mathscr{r}_2\tilde{C}_2^I \oplus \dots \oplus \mathscr{r}_n\tilde{C}_n^I$$
$= CBIFPWA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I)$□

**Remark 2.10.**According to the above property 3.9. the$CBIFPWA$ operator is a specific instance of the proposed $CBIFPDA$ operator. The $CBIFPDA$ operator is therefore more general than the $CBIFPWA$ operator.

**Property 2.11.**Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2, \dots, n)$ to be the collection of CBIFEs and $S(\tilde{C}_k^I) \neq 0, \forall\, k$, then we get $\lim_{(d_1,d_2,\dots,d_{n-1})\to(0,0,\dots,0)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = \frac{1}{n}(\tilde{C}_1^I \oplus \tilde{C}_2^I \oplus \dots \oplus \tilde{C}_n^I)$.

**Proof.** From given, $(d_1, d_2, \dots, d_{n-1}) \to (0,0, \dots,0)$, we get

$\mathrm{t}_k{}^{(d)} = \prod_{k=2}^{j-1}(S(\tilde{C}_k^I))^{d_k} = 1$and $\mathscr{r}_i{}^{(d)} = \frac{\mathrm{t}_i{}^{(d)}}{\sum_{k=1}^{n}\mathrm{t}_k{}^{(d)}} = \frac{1}{n}$. Then,

$$\lim_{(d_1,d_2,\dots,d_{n-1})\to(0,0,\dots,0)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = \frac{1}{n}\tilde{C}_1^I \oplus \frac{1}{n}\tilde{C}_2^I \oplus \dots \oplus \frac{1}{n}\tilde{C}_n^I$$
$= \frac{1}{n}(\tilde{C}_1^I \oplus \tilde{C}_2^I \oplus \dots \oplus \tilde{C}_n^I)$□

**Property 2.12.**Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2, \dots, n)$ to be the collection of CBIFEs and $S(\tilde{C}_1^I) \neq 0$ $or$ $S(\tilde{C}_1^I) \neq 1$, then we get $\lim_{d_1\to+\infty} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = \tilde{C}_1^I$.

**Proof.** Given that $d_1 \to +\infty$, for every $k = 2,3, \dots, n$, we get

$$\mathrm{t}_k{}^{(d)} = \prod_{k=1}^{j-1}(S(\tilde{C}_k^I))^{d_k} = (S(\tilde{C}_1^I))^{+\infty}(S(\tilde{C}_2^I))^{d_2} \dots \left(S(\tilde{C}_n^I)\right)^{d_n} = 0(\because 0 < S(\tilde{C}_1^I) < 1)$$

Then, $\sum_{k=1}^{n}\mathrm{t}_k{}^{(d)} = \mathrm{t}_1 = 1$ which implies $\mathscr{r}_1{}^{(d)} = \frac{\mathrm{t}_1{}^{(d)}}{\sum_{k=1}^{n}\mathrm{t}_k{}^{(d)}} = 1$ and $\mathscr{r}_j{}^{(d)} = 0$, for $j = 2,3, \dots, n$.

Hence, $\lim_{d_1\to+\infty} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = \tilde{C}_1^I$□

**Remark 2.13.**In contrast to the priority degrees of other CBIFNs, the priority degree $d_1$ of CBIFN is extremely large when $d_1 \to +\infty$. This suggests that $d_1$is highly significant. In this instance, $d_1$ determines the aggregate outcome obtained by the proposed $CBIFPDA$ operator.

**Property 2.14.**Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2, \dots, n)$ to be the collection of CBIFEs and $S(\tilde{C}_k^I) \neq 0\ \forall\, k = 1,2, \dots r + 1$ and $S(\tilde{C}_{r+1}^I) \neq 1$, then $\lim_{(d_1,d_2,\dots,d_r,d_{r+1})\to(0,0,\dots,0,+\infty)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = \frac{1}{r+1}(\tilde{C}_1^I \oplus \tilde{C}_2^I \oplus \dots \oplus \tilde{C}_{r+1}^I)$.

**Proof.** Considering that$(d_1, d_2, \dots, d_r, d_{r+1}) \to (0,0, \dots,0, +\infty)$, We have for every $k = 1,2, \dots r+1$,

$$\mathrm{t}_k{}^{(d)} = \prod_{j=1}^{k-1} (S(\tilde{C}_j^I))^{d_j} = (S(\tilde{C}_1^I))^{d_1} (S(\tilde{C}_2^I))^{d_2} \dots \left(S(\tilde{C}_{k-1}^I)\right)^{d_{k-1}}$$

$$\rightarrow (S(\tilde{C}_1^I))^0 (S(\tilde{C}_2^I))^0 \dots \left(S(\tilde{C}_{k-1}^I)\right)^0 = 1$$

Now, for $k = r+2, r+3, \dots, n$ we have

$$\mathrm{t}_k{}^{(d)} = \prod_{j=1}^{k-1} (S(\tilde{C}_j^I))^{d_j} = (S(\tilde{C}_1^I))^{d_1} (S(\tilde{C}_2^I))^{d_2} \dots \left(S(\tilde{C}_{k-1}^I)\right)^{d_{k-1}}$$

$$\rightarrow (S(\tilde{C}_1^I))^0 (S(\tilde{C}_2^I))^0 \dots \left(S(\tilde{C}_r^I)\right)^0 \left(S(\tilde{C}_{r+1}^I)\right)^{+\infty} \dots \left(S(\tilde{C}_{k-1}^I)\right)^{d_{k-1}} = 0$$

Then,for $k = 1,2, \dots r+1$, we get$\sum_{k=1}^{n} \mathrm{t}_k{}^{(d)} = r+1$ which implies $\mathcal{r}_i{}^{(d)} = \frac{\mathrm{t}_i{}^{(d)}}{\sum_{k=1}^{n} \mathrm{t}_k{}^{(d)}} = \frac{1}{r+1}$, andfor $k = r+2, r+3, \dots, n$, we get $\mathcal{r}_i{}^{(d)} = \frac{\mathrm{t}_i{}^{(d)}}{\sum_{k=1}^{n} \mathrm{t}_k{}^{(d)}} = \frac{0}{r+1} = 0$.

Hence,

$$\lim_{(d_1,d_2,\dots,d_r,d_{r+1}) \to (0,0,\dots,0,+\infty)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = \frac{1}{r+1} (\tilde{C}_1^I \oplus \tilde{C}_2^I \oplus \dots \oplus \tilde{C}_{r+1}^I) \square$$

**Remark 2.15.**If $(d_1, d_2, \dots, d_r, d_{r+1}) \to (0,0, \dots, 0, +\infty)$, then all of these CBIFNs $\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_{r+1}^I$ have a considerably higher priority than the CBIFNs $\tilde{C}_{r+2}^I, \tilde{C}_{r+3}^I, \dots, \tilde{C}_n^I$ and there is no prioritization link between them. Because of this, the CBIFNs $\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_{r+1}^I$ are the only factors that determines the aggregated value and they are given equal weightage in the aggregation process.

**Property 2.16.**Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\} (k = 1,2, \dots, n)$ to be the collection of CBIFEs and $S(\tilde{C}_{k+1}^I) \neq 1 \text{ or } 0$ then $\lim_{(d_1,d_2,\dots,d_r,d_{r+1}) \to (1,1,\dots,1,+\infty)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = CBIFPWA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_{r+1}^I)$.

**Proof.** Given $(d_1, d_2, \dots, d_r, d_{r+1}) \to (1,1, \dots, 1, +\infty)$, we have for every $k = 1,2, \dots, r+1$,

$$\mathrm{t}_k{}^{(d)} = \prod_{j=1}^{k-1} (S(\tilde{C}_j^I))^{d_j} = (S(\tilde{C}_1^I))^{d_1} (S(\tilde{C}_2^I))^{d_2} \dots \left(S(\tilde{C}_{k-1}^I)\right)^{d_{k-1}}$$

$$\rightarrow (S(\tilde{C}_1^I))^1 (S(\tilde{C}_2^I))^1 \dots \left(S(\tilde{C}_{k-1}^I)\right)^1 = \mathrm{t}_k$$

Now, for $k = r+2, r+3, \dots, n$ we have

$$\mathrm{t}_k{}^{(d)} = \prod_{j=1}^{k-1} (S(\tilde{C}_j^I))^{d_j} = (S(\tilde{C}_1^I))^{d_1} (S(\tilde{C}_2^I))^{d_2} \dots \left(S(\tilde{C}_{k-1}^I)\right)^{d_{k-1}}$$

$$\rightarrow (S(\tilde{C}_1^I))^1 (S(\tilde{C}_2^I))^1 \dots \left(S(\tilde{C}_r^I)\right)^1 \left(S(\tilde{C}_{r+1}^I)\right)^{+\infty} \dots \left(S(\tilde{C}_n^I)\right)^{d_n} = 0$$

Then,for all $k = 1,2, \dots, r+1$, we have$\sum_{k=1}^{n} \mathrm{t}_k{}^{(d)} \to \sum_{k=1}^{r+1} \mathrm{t}_k$ and $\mathcal{r}_i{}^{(d)} = \frac{\mathrm{t}_i{}^{(d)}}{\sum_{k=1}^{n} \mathrm{t}_k{}^{(d)}} \to \frac{\mathrm{t}_k}{\sum_{k=1}^{r+1} \mathrm{t}_k}$, andfor all $k = r+2, r+3 \dots, n$, we get $\mathcal{r}_i{}^{(d)} = \frac{\mathrm{t}_i{}^{(d)}}{\sum_{k=1}^{n} \mathrm{t}_k{}^{(d)}} \to \frac{0}{\sum_{k=1}^{r+1} \mathrm{t}_k} = 0$, .

Therefore,

$$\lim_{(d_1,d_2,\dots,d_r,d_{r+1}) \to (1,1,\dots,1,+\infty)} CBIFPDA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_n^I) = CBIFPWA(\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_{r+1}^I) \square$$

**Remark 2.17.**If $(d_1, d_2, \dots, d_r, d_{r+1}) \to (1,1, \dots, 1, +\infty)$, then all of these CBIFNs $\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_{r+1}^I$ have a considerably higher priority than the CBIFNs $\tilde{C}_{r+2}^I, \tilde{C}_{r+3}^I, \dots, \tilde{C}_n^I$ and there is a normal prioritization link between them. Because of this, the CBIFNs $\tilde{C}_1^I, \tilde{C}_2^I, \dots, \tilde{C}_{r+1}^I$ are the only factors that determines the aggregated value.

**CUBIC BIPOLAR INTUITIONISTIC FUZZY GEOMETRIC AOs WITH PRIORITY DEGREES:**

We now provide the ideas of the cubic bipolar intuitionistic fuzzy prioritized weighted geometric AOs and the concept of cubic bipolar intuitionistic fuzzy geometric AOs with priority degrees in the section that precedes.

**Definition 3.1.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu^{+l}_{\mathcal{B}^I k}, \mu^{+u}_{\mathcal{B}^I k}], [\mu^{-l}_{\mathcal{B}^I k}, \mu^{-u}_{\mathcal{B}^I k}], [\vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^{+u}_{\mathcal{B}^I k}], [\vartheta^{-l}_{\mathcal{B}^I k}, \vartheta^{-u}_{\mathcal{B}^I k}]\}, \{\mu^{+}_{\mathcal{B}^I k}, \mu^{-}_{\mathcal{B}^I k}, \vartheta^{+}_{\mathcal{B}^I k}, \vartheta^{-}_{\mathcal{B}^I k}\}\} (k = 1,2,\dots,n)$ to be the collection of CBIFEs. Then we defined the $CBIFPWG$ operator by the mapping $\mathcal{F} : \tilde{\mathcal{C}}_n^I \to \tilde{\mathcal{C}}^I$ as:

$CBIFPWG(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \tilde{\mathcal{C}}_1^{I\,r_1} \otimes \tilde{\mathcal{C}}_2^{I\,r_2} \otimes \dots \otimes \tilde{\mathcal{C}}_n^{I\,r_n}$ (7) where $r_i = \frac{t_i}{\sum_{k=1}^n t_k}$ and $t_j = \prod_{k=1}^{j-1} S(\tilde{\mathcal{C}}_k^I)$, $j = 2,3,\dots,n$, where $S(\tilde{\mathcal{C}}_k^I)$ be the score function of the $k^{th}$ $CBIFN$ and $t_1 = 1$.

**Definition 3.2.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu^{+l}_{\mathcal{B}^I k}, \mu^{+u}_{\mathcal{B}^I k}], [\mu^{-l}_{\mathcal{B}^I k}, \mu^{-u}_{\mathcal{B}^I k}], [\vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^{+u}_{\mathcal{B}^I k}], [\vartheta^{-l}_{\mathcal{B}^I k}, \vartheta^{-u}_{\mathcal{B}^I k}]\}, \{\mu^{+}_{\mathcal{B}^I k}, \mu^{-}_{\mathcal{B}^I k}, \vartheta^{+}_{\mathcal{B}^I k}, \vartheta^{-}_{\mathcal{B}^I k}\}\} (k = 1,2,\dots,n)$ to be the collection of CBIFEs. Then we defined the $CBIFPDG$ operator by the mapping $\mathcal{F} : \tilde{\mathcal{C}}_n^I \to \tilde{\mathcal{C}}^I$ as:

$CBIFPDG(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \tilde{\mathcal{C}}_1^{I\,r_1^{(d)}} \otimes \tilde{\mathcal{C}}_2^{I\,r_2^{(d)}} \otimes \dots \otimes \tilde{\mathcal{C}}_n^{I\,r_n^{(d)}}$ (8) where $r_i^{(d)} = \frac{t_i^{(d)}}{\sum_{k=1}^n t_k^{(d)}}$ and $t_k^{(d)} = \prod_{k=1}^{j-1} (S(\tilde{\mathcal{C}}_k^I))^{d_k}$, $j = 2,3,\dots,n$, where $S(\tilde{\mathcal{C}}_k^I)$ be the score function of the $k^{th}$ $CBIFN$ and $t_1 = 1$.

**Theorem 3.3.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu^{+l}_{\mathcal{B}^I k}, \mu^{+u}_{\mathcal{B}^I k}], [\mu^{-l}_{\mathcal{B}^I k}, \mu^{-u}_{\mathcal{B}^I k}], [\vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^{+u}_{\mathcal{B}^I k}], [\vartheta^{-l}_{\mathcal{B}^I k}, \vartheta^{-u}_{\mathcal{B}^I k}]\}, \{\mu^{+}_{\mathcal{B}^I k}, \mu^{-}_{\mathcal{B}^I k}, \vartheta^{+}_{\mathcal{B}^I k}, \vartheta^{-}_{\mathcal{B}^I k}\}\} (k = 1,2,\dots,n)$ to be the collection of CBIFEs. Then we defined the $CBIFPDG$ operator by the mapping $\mathcal{F} : \tilde{\mathcal{C}}_n^I \to \tilde{\mathcal{C}}^I$ as:

$$CBIFPDA(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \tilde{\mathcal{C}}_1^{I\,r_1^{(d)}} \otimes \tilde{\mathcal{C}}_2^{I\,r_2^{(d)}} \otimes \dots \otimes \tilde{\mathcal{C}}_n^{I\,r_n^{(d)}}$$

$$= \{\{[\prod_{k=1}^{n} (\mu^{+l}_{\mathcal{B}^I k})^{r_k^{(d)}}, \prod_{k=1}^{n} (\mu^{+u}_{\mathcal{B}^I k})^{r_k^{(d)}}], [-(1 - \prod_{k=1}^{n} (1 - (-\mu^{-l}_{\mathcal{B}^I k}))^{r_k^{(d)}}), -(1 - \prod_{k=1}^{n} (1 - (-\mu^{-u}_{\mathcal{B}^I k}))^{r_k^{(d)}})], [1 - \prod_{k=1}^{n} (1 - \vartheta^{+l}_{\mathcal{B}^I k})^{r_k^{(d)}}, 1 - \prod_{k=1}^{n} (1 - \vartheta^{+u}_{\mathcal{B}^I k})^{r_k^{(d)}}], [-(\prod_{k=1}^{n} (\vartheta^{-l}_{\mathcal{B}^I k})^{r_k^{(d)}}), -(\prod_{k=1}^{n} (\vartheta^{-u}_{\mathcal{B}^I k})^{r_k^{(d)}})]\},$$
$$\{\prod_{k=1}^{n} (\mu^{+}_{\mathcal{B}^I k})^{r_k^{(d)}}, -\left(1 - \prod_{k=1}^{n} (1 - (-\mu^{-}_{\mathcal{B}^I k}))^{r_k^{(d)}}\right), 1 - \prod_{k=1}^{n} (1 - \vartheta^{+}_{\mathcal{B}^I k})^{r_k^{(d)}}, -(\prod_{k=1}^{n} (\vartheta^{-}_{\mathcal{B}^I k})^{r_k^{(d)}})\}\} \quad (9)$$

**Proof.** The proof is comparable to Theorem 2.3.□

**Theorem 3.4.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu^{+l}_{\mathcal{B}^I k}, \mu^{+u}_{\mathcal{B}^I k}], [\mu^{-l}_{\mathcal{B}^I k}, \mu^{-u}_{\mathcal{B}^I k}], [\vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^{+u}_{\mathcal{B}^I k}], [\vartheta^{-l}_{\mathcal{B}^I k}, \vartheta^{-u}_{\mathcal{B}^I k}]\}, \{\mu^{+}_{\mathcal{B}^I k}, \mu^{-}_{\mathcal{B}^I k}, \vartheta^{+}_{\mathcal{B}^I k}, \vartheta^{-}_{\mathcal{B}^I k}\}\} (k = 1,2,\dots,n)$ as the collection of CBIFEs. Then, the $CBIFPDG$ operator encounters the idempotency property (i.e) if $\tilde{\mathcal{C}}_k^I = \tilde{\mathcal{C}}^I, \forall k = 1,2,\dots,n$ we have $CBIFPDG(\tilde{\mathcal{C}}_1^I, \tilde{\mathcal{C}}_2^I, \dots, \tilde{\mathcal{C}}_n^I) = \tilde{\mathcal{C}}^I$.

**Proof.** The result is comparable to Theorem 2.5.□

**Theorem 3.5.** Consider $\tilde{\mathcal{C}}_k^I = \{\{[\mu^{+l}_{\mathcal{B}^I k}, \mu^{+u}_{\mathcal{B}^I k}], [\mu^{-l}_{\mathcal{B}^I k}, \mu^{-u}_{\mathcal{B}^I k}], [\vartheta^{+l}_{\mathcal{B}^I k}, \vartheta^{+u}_{\mathcal{B}^I k}], [\vartheta^{-l}_{\mathcal{B}^I k}, \vartheta^{-u}_{\mathcal{B}^I k}]\}, \{\mu^{+}_{\mathcal{B}^I k}, \mu^{-}_{\mathcal{B}^I k}, \vartheta^{+}_{\mathcal{B}^I k}, \vartheta^{-}_{\mathcal{B}^I k}\}\}$ $(k = 1,2,\dots,n)$ as the collection of CBIFEs. Then, the $CBIFPDG$ operator meets the monotonicity property (i.e.) if $\tilde{C}_k^I \leq \tilde{C}_k^{I'}, \forall k = 1,2,\ldots,n$, we get $CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) \leq CBIFPDG(\tilde{C}_1^{I'}, \tilde{C}_2^{I'}, \ldots, \tilde{C}_n^{I'})$.

**Proof.** The proof is equivalent to the Theorem 2.6.□

**Theorem 3.6.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\ldots,n)$to be the collection of CBIFEs. Then, the $CBIFPDG$ operator satisfies the boundedness property.

**Proof.** The proof is in contrast to Theorem 2.7.□

**Theorem 3.7.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\ldots,n)$ and $\tilde{C}_k^{I'} = \{\{[\mu_{\mathcal{B}^{I'} k}^{+l}, \mu_{\mathcal{B}^{I'} k}^{+u}], [\mu_{\mathcal{B}^{I'} k}^{-l}, \mu_{\mathcal{B}^{I'} k}^{-u}], [\vartheta_{\mathcal{B}^{I'} k}^{+l}, \vartheta_{\mathcal{B}^{I'} k}^{+u}], [\vartheta_{\mathcal{B}^{I'} k}^{-l}, \vartheta_{\mathcal{B}^{I'} k}^{-u}]\}, \{\mu_{\mathcal{B}^{I'} k}^{+}, \mu_{\mathcal{B}^{I'} k}^{-}, \vartheta_{\mathcal{B}^{I'} k}^{+}, \vartheta_{\mathcal{B}^{I'} k}^{-}\}\}$ $(k = 1,2,\ldots,n)$ be the two assemblage of CBIFEs and let $\varphi > 0$, then we have the following properties:

1. $CBIFPDG(\tilde{C}_1^I \otimes \tilde{C}_1^{I'}, \tilde{C}_2^I \otimes \tilde{C}_1^{I'}, \ldots, \tilde{C}_n^I \otimes \tilde{C}_1^{I'}) = CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) \oplus \tilde{C}_1^{I'}$
2. $CBIFPDG(\varphi\tilde{C}_1^I, \varphi\tilde{C}_2^I, \ldots, \varphi\tilde{C}_n^I) = \varphi\, CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I)$
3. $CBIFPDG(\varphi\tilde{C}_1^I \otimes \tilde{C}_1^{I'}, \varphi\tilde{C}_2^I \otimes \tilde{C}_1^{I'}, \ldots, \varphi\tilde{C}_n^I \otimes \tilde{C}_1^{I'}) = \varphi CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) \otimes \tilde{C}_1^{I'}$
4. $CBIFPDG(\tilde{C}_1^I \otimes \tilde{C}_1^{I'}, \tilde{C}_2^I \otimes \tilde{C}_2^{I'}, \ldots, \tilde{C}_n^I \otimes \tilde{C}_n^{I'}) = CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) \otimes CBIFPDG(\tilde{C}_1^{I'}, \tilde{C}_2^{I'}, \ldots, \tilde{C}_n^{I'})$

**Proof.** Proofs are trivial by the definition 3.2.□

**Property 3.8.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\ldots,n)$to be the collection of CBIFEs. Then, $\lim_{(d_1,d_2,\ldots,d_{n-1})\to(1,1,\ldots,1)} CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) = CBIFPWG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I)$.

**Property 3.9.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\ldots,n)$ to be the collection of CBIFEs and $S(\tilde{C}_k^I) \neq 0, \forall k$, then we get $\lim_{(d_1,d_2,\ldots,d_{n-1})\to(0,0,\ldots,0)} CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) = \frac{1}{n}(\tilde{C}_1^I \otimes \tilde{C}_2^I \otimes \ldots \otimes \tilde{C}_n^I)$.

**Property 3.10.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\ldots,n)$ to be the collection of CBIFEs and $S(\tilde{C}_1^I) \neq 0 \text{ or } S(\tilde{C}_1^I) \neq 1$, then we get $\lim_{d_1\to+\infty} CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) = \tilde{C}_1^I$.

**Property 3.11.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\ldots,n)$ to be the collection of CBIFEs and $S(\tilde{C}_k^I) \neq 0\ \forall\ k = 1,2,\ldots r+1$ and $S(\tilde{C}_{r+1}^I) \neq 1$, then $\lim_{(d_1,d_2,\ldots,d_r,d_{r+1})\to(0,0,\ldots,0,+\infty)} CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) = \frac{1}{r+1}(\tilde{C}_1^I \otimes \tilde{C}_2^I \otimes \ldots \otimes \tilde{C}_{r+1}^I)$.

**Property 3.12.** Consider $\tilde{C}_k^I = \{\{[\mu_{\mathcal{B}^I k}^{+l}, \mu_{\mathcal{B}^I k}^{+u}], [\mu_{\mathcal{B}^I k}^{-l}, \mu_{\mathcal{B}^I k}^{-u}], [\vartheta_{\mathcal{B}^I k}^{+l}, \vartheta_{\mathcal{B}^I k}^{+u}], [\vartheta_{\mathcal{B}^I k}^{-l}, \vartheta_{\mathcal{B}^I k}^{-u}]\}, \{\mu_{\mathcal{B}^I k}^{+}, \mu_{\mathcal{B}^I k}^{-}, \vartheta_{\mathcal{B}^I k}^{+}, \vartheta_{\mathcal{B}^I k}^{-}\}\}$ $(k = 1,2,\ldots,n)$ to be the collection of CBIFEs and $S(\tilde{C}_{k+1}^I) \neq 1 \text{ or } 0$ then $\lim_{(d_1,d_2,\ldots,d_r,d_{r+1})\to(1,1,\ldots,1,+\infty)} CBIFPDG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_n^I) = CBIFPWG(\tilde{C}_1^I, \tilde{C}_2^I, \ldots, \tilde{C}_{r+1}^I)$.

## MULTI-CRITERIA DECISION MAKING USING PROPOSED AGGREGATION OPERATORS

Making decisions is a crucial process that allows one to select the most logical option from the alternatives that are available. By assembling pertinent information and presenting potential outcomes, a decision making process might assists us in reaching more deliberate, thoughtful conclusions. In addition, MCDM have also employed aggregation operators withpriority degrees. These operators are regarded as an additional helpful and precise tool for ranking the choices that exist. In order to address aMCDM problem, we employ the aggregation operators with priority degrees on cubic bipolar intuitionistic fuzzy sets.

Consider $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \dots \mathfrak{x}_m\}$to be the collection of alternatives and $\mathcal{P} = \{\wp_1, \wp_2, \dots, \wp_n\}$ be the collection of parameters or attributes. Strict priority relation is assigned between the parameters. $\wp_1 \geq_{d_1} \wp_2 \geq_{d_2} \wp_3 \dots \geq_{d_{n-1}} \wp_n$shows that parameter $\wp_i$ has high priority than $\wp_{i+1}$ with priority degree $d_i, i \in \{1,2, \dots n-1\}$. Let $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_k\}$ be the assemblage of decision makers and, $\mathcal{D}_1 \geq_{d'_1} \mathcal{D}_2 \geq_{d'_2} \mathcal{D}_3 \dots \geq_{d'_{n-1}} \mathcal{D}_n$ be the priorities assigned between the decision makers. Decision makers $\mathcal{D}_k$ provide a matrix $\mathbb{D}^{(k)} = (B_{ij}^{(k)})_{m\times n}$based on the DMs personal opinions for the alternative $\mathfrak{x}_i$ and parameter$\wp_j$.

In this part, we put forward the approach of aggregation operators together with thepriority degrees for resolving the MCDM problem in the layout of cubic bipolar intuitionistic fuzzy sets.

Here is the procedure for solving the MCDM problem.

**Step 1.** Acquire the decision matrix $\mathbb{D}^{(k)} = (B_{ij}^{(k)})_{m\times n}$ in which all the entries represents a CBIFNs that the decision makers have assigned based on their perspectives.

**Step 2.** While making decisions, benefits and costs are the two primary factors that we often taken into account. In MCDM, success is achieved through maximizing the benefit parameter and minimizing the cost parameter. Normalization is not required if all the parameters are of same kind. Using the normalizing formula, we convert the matrix $\mathbb{D}^{(k)}$ into a normalized matrix as,

$$\mathcal{N}^{(k)} = (G_{ij}^{(k)})_{m\times n} = \begin{cases} (B_{ij}^{(k)})^c \,; for\ value\ of\ loss\ parameter \\ B_{ij}^{(k)}; for\ value\ of\ benefit\ parameter \end{cases}$$

where$(B_{ij}^{(k)})^c$ indicates the complement of $B_{ij}^{(k)}$.

**Step 3.**Utilizing one of the aforementioned aggregation operators, accumulate all the independent CBIF decision matrices of the alternatives$\mathcal{N}^{(k)} = (G_{ij}^{(k)})_{m\times n}$ into a single evaluation matrix.

**Step 4.** Aggregate the CBIFNs for each of the alternatives using CBIFPDA (or CBIFPDG) operators.

**Step 5.** Evaluate the score for each accumulative CBIFNs.

**Step 6.** The best option was chosen after the alternatives were categorized using the scoring function.

**Example 4.1.**

Let's say a school management wants to select the expertise teacher for higher secondary and the management committee made up of three professionals/specialists/Decision makers $\mathcal{D}_1, \mathcal{D}_2$and $\mathcal{D}_3$ with the goal of choosing the best teacher among the four candidates $\mathfrak{X} = \{\mathfrak{x}_1, \mathfrak{x}_2, \mathfrak{x}_3, \mathfrak{x}_4\}$attending the interview. The specialists think about certain parameters which includes $\wp_1$ =Communication skills, $\wp_2$ =Pedagogical expertise, $\wp_3$ = Adaptability. In accordance with criteria's$\wp_j, j = 1,2,3$, the decision makers generate decision matrices for alternatives$\mathfrak{x}_i, i = 1,2,3,4$ as shown below:

**Step 1.** Consider the decision matrices for the decision makers $\mathcal{D}_i, i = 1,2,3$ are obtained in the Tables 4.1, 4.2 and 4.3.

**TABLE 4.1.**Decision matrix $\mathbb{D}^{(1)}$ by the decision maker $\mathcal{D}_1$

| $\mathfrak{X}/\mathcal{P}$ | $\wp_1$ | $\wp_2$ | $\wp_3$ |
|---|---|---|---|
| $\mathfrak{x}_1$ | $\{\{[0.16,0.25],[-0.23,-0.18],[0.21,0.30],[-0.25,-0.19]\},\{0.18,-0.20,0.26,-0.23\}\}$ | $\{\{[0.55,0.60],[-0.58,-0.51],[0.25,0.29],[-0.27,-0.21]\},\{0.59,-0.56,0.24,-0.25\}\}$ | $\{\{[0.41,0.49],[-0.48,-0.39],[0.36,0.43],[-0.41,-0.35]\},\{0.47,-0.40,0.38,-0.36\}\}$ |
| $\mathfrak{x}_2$ | $\{\{[0.35,0.43],[-0.38,-0.29],$ | $\{\{[0.63,0.69],[-0.65,-0.57],$ | $\{\{[0.22,0.28],[-0.26,-0.18],$ |

| | | | |
|---|---|---|---|
| | [0.40,0.49], [−0.46, −0.39]}, {0.38, −0.32,0.45, −0.43}} | [0.25,0.31], [−0.28, −0.22]}, {0.67, −0.59,0.27, −0.23}} | [0.14,0.23], [−0.15, −0.11]}, {0.25, −0.19,0.21, −0.13}} |
| $\mathfrak{x}_3$ | {{[0.54,0.57], [−0.58, −0.51], [0.56,0.62], [−0.63, −0.59]}, {0.55, −0.53,0.60, −0.58}} | {{[0.14,0.22], [−0.19, −0.15], [0.34,0.43], [−0.38, −0.33]}, {0.18, −0.17,0.41, −0.35}} | {{[0.17,0.26], [−0.31, −0.26], [0.43,0.51], [−0.53, −0.57]}, {0.23, −0.28,0.45, −0.55}} |
| $\mathfrak{x}_4$ | {{[0.63,0.67], [−0.60, −0.55], [0.25,0.29], [−0.26, −0.19]}, {0.65, −0.58,0.27, −0.23}} | {{[0.24,0.30], [−0.28, −0.25], [0.46,0.53], [−0.51, −0.49]}, {0.26, −0.27,0.48, −0.50}} | {{[0.55,0.60], [−0.57, −0.52], [0.37,0.39], [−0.32, −0.28]}, {0.57, −0.55,0.38, −0.30}} |

**TABLE 4.2.**Decision matrix $\mathbb{D}^{(2)}$by the decision maker $\mathcal{D}_2$

| $\mathfrak{X}$ / $\mathcal{P}$ | $\wp_1$ | $\wp_2$ | $\wp_3$ |
|---|---|---|---|
| $\mathfrak{x}_1$ | {{[0.12,0.24], [−0.21, −0.15], [0.34,0.42], [−0.39, −0.33]}, {0.17, −0.18,0.38, −0.35}} | {{[0.10,0.27], [−0.25, −0.11], [0.23,0.34], [−0.35, −0.27]}, {0.20, −0.18,0.32, −0.29}} | {{[0.53,0.58], [−0.52, −0.47], [0.38,0.41], [−0.37, −0.33]}, {0.55, −0.51,0.40, −0.35}} |
| $\mathfrak{x}_2$ | {{[0.45,0.52], [−0.50, −0.43], [0.26,0.33], [−0.34, −0.28]}, {0.48, −0.45,0.29, −0.32}} | {{[0.54,0.59], [−0.43, −0.39], [0.32,0.37], [−0.38, −0.35]}, {0.58, −0.43,0.33, −0.34}} | {{[0.17,0.26], [−0.30, −0.24], [0.31,0.39], [−0.35, −0.29]}, {0.21, −0.27,0.34, −0.31}} |
| $\mathfrak{x}_3$ | {{[0.65,0.69], [−0.68, −0.62], [0.23,0.30], [−0.36, −0.31]}, {0.66, −0.65,0.29, −0.34}} | {{[0.47,0.53], [−0.55, −0.48], [0.42,0.46], [−0.43, −0.38]}, {0.50, −0.49,0.43, −0.40}} | {{[0.24,0.28], [−0.29, −0.18], [0.46,0.53], [−0.55, −0.51]}, {0.26, −0.20,0.48, −0.53}} |
| $\mathfrak{x}_4$ | {{[0.36,0.42], [−0.40, −0.33], [0.47,0.49], [−0.45, −0.43]}, {0.38, −0.35,0.48, −0.42}} | {{[0.23,0.29], [−0.28, −0.22], [0.41,0.44], [−0.45, −0.38]}, {0.25, −0.27,0.43, −0.40}} | {{[0.38,0.43], [−0.42, −0.40], [0.51,0.53], [−0.50, −0.45]}, {0.39, −0.41,0.52, −0.49}} |

**TABLE4.3.**Decision matrix $\mathbb{D}^{(3)}$by the decision maker $\mathcal{D}_3$

| $\mathfrak{X}$ / $\mathcal{P}$ | $\wp_1$ | $\wp_2$ | $\wp_3$ |
|---|---|---|---|
| $\mathfrak{x}_1$ | {{[0.15,0.20], [−0.35, −0.30], [0.45,0.50], [−0.25, −0.20]}, {0.17, −0.32,0.47, −0.22}} | {{[0.47,0.50], [−0.55, −0.49], [0.42,0.45], [−0.43, −0.37]}, {0.49, −0.51,0.43, −0.40}} | {{[0.38,0.42], [−0.41, −0.39], [0.51,0.54], [−0.38, −0.35]}, {0.39, −0.40,0.52, −0.36}} |
| $\mathfrak{x}_2$ | {{[0.37,0.45], [−0.28, −0.23], [0.39,0.48], [−0.56, −0.49]}, {0.40, −0.25,0.45, −0.53}} | {{[0.17,0.23], [−0.30, −0.27], [0.39,0.43], [−0.31, −0.25]}, {0.22, −0.29,0.41, −0.27}} | {{[0.23,0.28], [−0.33, −0.30], [0.53,0.58], [−0.63, −0.60]}, {0.25, −0.31,0.55, −0.61}} |
| $\mathfrak{x}_3$ | {{[0.10,0.12], [−0.18, −0.15], [0.45,0.58], [−0.33, −0.26]}, {0.11, −0.16,0.55, −0.29}} | {{[0.61,0.66], [−0.70, −0.65], [0.25,0.29], [−0.10, −0.03]}, {0.63, −0.68,0.27, −0.09}} | {{[0.11,0.17], [−0.22, −0.18], [0.44,0.66], [−0.33, −0.28]}, {0.13, −0.20,0.55, −0.30}} |
| $\mathfrak{x}_4$ | {{[0.60,0.63], [−0.54, −0.50], [0.10,0.17], [−0.23, −0.19]}, {0.61, −0.52,0.14, −0.20}} | {{[0.14,0.25], [−0.19, −0.16], [0.44,0.55], [−0.43, −0.37]}, {0.18, −0.17,0.50, −0.40}} | {{[0.35,0.42], [−0.38, −0.28], [0.40,0.48], [−0.46, −0.38]}, {0.37, −0.31,0.44, −0.42}} |

**Step 2.** Normalization is not essential because every parameters are of benefit type.

**Step 3.** Now, take the priority degrees as $(d_1', d_2', d_3') = (1,1,1)$ and by using the proposed $CBIFPDA$ aggregationoperator, we accumulate all the independent $CBIF$ decision matrices into a single evaluation matrix is shown in Table 4.4.

Before that, we find $\mathrm{t}_{ij}{}^{(1)}, \mathrm{t}_{ij}{}^{(2)}$ and $\mathrm{t}_{ij}{}^{(3)}$, which has been used in the calculation of $CBIFPDA$ operator.

$$\mathrm{t}_{ij}{}^{(1)} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \mathrm{t}_{ij}{}^{(2)} = \begin{pmatrix} 0.4900 & 0.5033 & 0.5042 \\ 0.5092 & 0.5067 & 0.4942 \\ 0.5050 & 0.4925 & 0.5058 \\ 0.5075 & 0.5025 & 0.4867 \end{pmatrix}$$

$$\mathrm{t}_{ij}{}^{(3)} = \begin{pmatrix} 0.2417 & 0.2538 & 0.2530 \\ 0.2601 & 0.2749 & 0.2364 \\ 0.2626 & 0.2413 & 0.2626 \\ 0.2512 & 0.2491 & 0.2373 \end{pmatrix}$$

**TABLE 4.4.** Aggregated decision matrix

| $\mathfrak{X}/\mathcal{P}$ | $\wp_1$ | $\wp_2$ |
|---|---|---|
| $\mathfrak{x}_1$ | {{[0.1475,0.2404], [−0.2377, −0.1836], [0.2677,0.3544], [−0.2926, −0.2337]}, {0.1758, −0.2073,0.3144, −0.2647}} | {{[0.4380,0.5092], [−0.4523, −0.3268], [0.2631,0.3234], [−0.3187, −0.2525]}, {0.4876, −0.3991,0.2835, −0.2851}} |
| $\mathfrak{x}_2$ | {{[0.3834,0.4604], [−0.3932, −0.3139], [0.3502,0.4360], [−0.4449, −0.3768]}, {0.4134, −0.3404,0.3966, −0.4171}} | {{[0.5541,0.6138], [−0.5130, −0.4560], [0.2872,0.3429], [−0.3145, −0.2639]}, {0.5964, −0.4832,0.3049, −0.2691}} |
| $\mathfrak{x}_3$ | {{[0.5299,0.5644], [−0.5101, −0.4496], [0.4204,0.4989], [−0.5274, −0.4806]}, {0.5403, −0.4702,0.4812, −0.4833}} | {{[0.3286,0.3983], [−0.3081, −0.2560], [0.3459,0.4149], [−0.3624, −0.3100]}, {0.3622, −0.2785,0.3921, −0.3342}} |
| $\mathfrak{x}_4$ | {{[0.5618,0.6053], [−0.5258, −0.4682], [0.2632,0.3126], [−0.3169, −0.2681]}, {0.5808, −0.4936,0.2902, −0.2866}} | {{[0.2236,0.2902], [−0.2650, −0.2262], [0.4423,0.5051], [−0.4825, −0.4442]}, {0.2462, −0.2528,0.4678, −0.4593}} |

| $\mathfrak{X}/\mathcal{P}$ | $\wp_3$ |
|---|---|
| $\mathfrak{x}_1$ | {{[0.4433,0.5086], [−0.4801, −0.4115], [0.3844,0.4383], [−0.3945, −0.3443]}, {0.4840, −0.4289,0.4034, −0.3572}} |
| $\mathfrak{x}_2$ | {{[0.2074,0.2743], [−0.2798, −0.2095], [0.2107,0.3034], [−0.2972, −0.2520]}, {0.2388, −0.2246,0.2748, −0.2703}} |
| $\mathfrak{x}_3$ | {{[0.1822,0.2532], [−0.2890, −0.2216], [0.4399,0.5358], [−0.5107, −0.5181]}, {0.2248, −0.2419,0.4722, −0.5135}} |
| $\mathfrak{x}_4$ | {{[0.4818,0.5347], [−0.4945, −0.4434], [0.4095,0.4376], [−0.3960, −0.3463]}, {0.4998, −0.4678,0.4236, −0.3762}} |

**Step 4.** We now aggregate the above $CBIFNs$ for each parameters $\wp_j$ by the proposed $CBIFPDA$ operator under the priority degree $(\wp_1, \wp_2, \wp_3) = (1,1,1)$, for that, we have

$$\mathrm{t}_{ij} = \begin{pmatrix} 1 & 0.4825 & 0.2510 \\ 1 & 0.5200 & 0.2708 \\ 1 & 0.5246 & 0.2667 \\ 1 & 0.5222 & 0.2605 \end{pmatrix}$$

We get the accumulative decision matrix as provide in Table 4.5.

**TABLE 4.5.** Accumulative decision matrix

| Parameters | $CBIF$ values |
|---|---|

| | |
|---|---|
| $\mathfrak{x}_1$ | $\{\{[0.2863,0.3684],[-0.3148,-0.2423],[0.2807,0.3563],[-0.3155,-0.2560]\},$ $\{0.3252,-0.2764,0.3167,-0.2845\}\}$ |
| $\mathfrak{x}_2$ | $\{\{[0.4172,0.4880],[-0.4036,-0.3292],[0.3070,0.3849],[-0.3883,-0.3276]\},$ $\{0.4528,-0.3540,0.3475,-0.3559\}\}$ |
| $\mathfrak{x}_3$ | $\{\{[0.4334,0.4812],[-0.4044,-0.3431],[0.3997,0.4777],[-0.4814,-0.4418]\},$ $\{0.4531,-0.3654,0.4519,-0.4485\}\}$ |
| $\mathfrak{x}_4$ | $\{\{[0.4690,0.5199],[-0.4263,-0.3753],[0.3268,0.3779],[-0.3815,-0.3358]\},$ $\{0.4892,-0.4026,0.3527,-0.3550\}\}$ |

**Step 5.** Compute the score for each $CBIF$ values.

$$S(\mathfrak{x}_1) = 0.50407$$
$$S(\mathfrak{x}_2) = 0.52529$$
$$S(\mathfrak{x}_3) = 0.52475$$
$$S(\mathfrak{x}_4) = 0.52405$$

**Step 6.** We order the alternatives in accordance to the score values as,

$$\mathfrak{x}_2 > \mathfrak{x}_3 > \mathfrak{x}_1 > \mathfrak{x}_4$$

We conclude that $\mathfrak{x}_2$ is the best alternatives out of all the alternatives.

**CONCLUSION :**

Here, we focused onthe cubic bipolar intuitionistic fuzzy sets by generalizing theBIFSin this work. Priority degree theories will facilitate the fusion of large-scale of CBIFdata. Furthermore, a numerical instance of MCDM in accordance with CBIF set to deal with bipolar uncertainties under priority degree orders are given.

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